

4.3

$$4\pi R^2 |\vec{E}| = \frac{1}{\epsilon_0} \int \rho(\vec{r}) d\tau \quad \rho \propto r \Rightarrow \rho = \begin{cases} kr & \text{for } r \leq R \\ 0 & \text{otherwise} \end{cases}$$

$$4\pi R^2 E = \frac{k}{\epsilon_0} \int r^3 \sin\theta dr d\phi d\theta = \frac{k}{4\epsilon_0} \cdot 4\pi R^4 = \frac{\pi k R^4}{\epsilon_0} = \frac{q_{tot}}{\epsilon_0} \Rightarrow q_{tot} = \pi k R^4$$

$$\Rightarrow k = \frac{q_{tot}}{\pi R^4}; \quad q_{enc} = \pi k r^4 = q_{tot} \left(\frac{r}{R}\right)^4 \epsilon_0$$

$$\Rightarrow \vec{E} = \frac{q_{enc}}{4\pi\epsilon_0 r^2} \hat{r} = \frac{q_{tot}}{4\pi\epsilon_0 R^4} r^2 \hat{r}$$

The nucleus & cloud are separated by distance d ,

creating the dipole $\vec{p} = q_{tot} \vec{d}$, from an external electric field, \vec{E}_{ext}

$$\text{In this case, } E_{ext} = \frac{q_{tot}}{4\pi\epsilon_0 R^4} d^2 = \frac{p d}{4\pi\epsilon_0 R^4} = \frac{p^2}{4\pi\epsilon_0 q_{tot} R^4}$$

$$\Rightarrow p = \sqrt{4\pi\epsilon_0 q_{tot} R^4 E}$$